Interpretation of Static-Weighting $\mathcal{H}$-Infinity Design Approaches for Interval Plants

Baowei Ji[1], Haniph A. Latchman[1], and Oscar D. Crisalle[2]

Abstract

This paper takes advantage of the critical direction theory to select a meaningful static weight for the H_∞ design of robust controller for parametrically uncertain plants. The mechanisms leading to conservatism in an earlier approach utilizing a constant overbounding uncertainty bound are analyzed first; then, an alternative less conservative uncertainty bound is presented. Finally, an example is used to illustrate that the proposed approach can substantially increase the stabilizable uncertainty level.

1 Introduction

Results on stability analysis for systems with parametric uncertainties have emerged steadily over the years, including outstanding results such as the edge theorem, the mapping theorem, the generalized Kharitonov theorem [2], the critical directional theory (CDT) [9], and many others. However, robust control synthesis for interval plants remains a relatively unsolved problem for which only few results are available. An early attempt to synthesize a robust controller was made by Keel *et al.* [8], where interval plants are overbounded by a constant uncertainty disk in the frequency domain. By assigning such a constant bound as the weighting function, standard results from the H_∞ synthesis theory can be applied to solve for a robustly stabilizing controller. This approach shows great promise because it is quite simple and makes effective use of the well-developed H_∞ synthesis methodology. To facilitate comparison with the alternative results developed in this paper, we refer to the method in [8] as the *maximum perturbation radius* (MPR) approach. Unfortunately, the MPR weighting strategy may introduce conservatism since the overbounding operation guarantees only sufficient conditions for robust stability. Other researchers have tried to attack the synthesis problem using fixed-structure controllers such as PID, or other low-order controllers [3] [6] [7]. Yet, it can be argued that these approaches suffer from inherent shortcomings by restricting attention to only fixed-structure controllers.

Motivated by the fact that the *critical direction theory* [1] [9] gives necessary and sufficient stability conditions that involve a critical perturbation radius (CPR), the conservatism of the MPR weighting is explored in detail in this paper. Furthermore, the critical direction theory is applied to introduce an *exact* weighting strategy that results in an even larger stability margin than that based on the MPR weighting method. The effective critical perturbation radius(ECPR) weighting strategy proposed in this paper recovers the simplicity of the MPR weighting while avoiding the conservatism induced by uncertainty over-bounding.

2 Problem Formulation

Consider a SISO linear uncertain plant $p(s)$ in the forward path of a feedback system including a controller $c(s)$ as shown in Fig. 1. The plant family $p(s) = p_0(s) + \delta(s)$ includes a known nominal SISO system $p_0(s)$, and a perturbation $\delta(s)$ characterized by a set of real parameters, namely, the coefficients of the numerator and denominator polynomials. Each parameter in turn is an element of a real interval of width 2ϵ centered about the nominal value of the corresponding parameter of the nominal plant $p_0(s)$. Therefore, the uncertainty description for the problem is a real hypercube $\Delta(\epsilon)$ with sides of width 2ϵ. The value ϵ is a non-negative scalar that represents the level of uncertainty. In general, system $p_0(s)$ is neither open-loop stable nor unity-negative-feedback system stable. A *stabilizing* controller $c(s)$ needs to be designed so that the closed-loop system of Fig. 1 is robustly stable, *i.e.*, stable over all uncertain plants $p(s)$.

It is assumed that the set of allowable perturbations $\Delta(\epsilon)$ is such that systems $p(s)$ and $p_0(s)$ have the same number of open-loop unstable poles.

This assumption can be checked by the following steps: (1) Use the Routh-Hurwitz criterion for the denominator Kharitonov polynomials of $p(s)$ to find an uncertainty level ϵ_1 such that the number of unstable roots of the family of denominator-polynomials of $p(s)$ does not change for all $\epsilon \leq \epsilon_1$ [8]. (2) Use the *edge theorem*

[1]B. Ji and H. A. Latchman are with the Department of Electrical & Computer Engineering, University of Florida, Gainesville, FL 32611 USA (e-mail: baowei@ufl.edu; latchman@list.ufl.edu).

[2]O. D. Crisalle is with the Department of Chemical Engineering, University of Florida, Gainesville, FL 32611 USA (e-mail: crisalle@che.ufl.edu).

to find another uncertainty level ϵ_2 such that there is no unstable pole-zero cancellation in the family of $p(s)$ for all $\epsilon \leq \epsilon_2$ [4]. (3) Define the upper bound of the uncertainty as $\epsilon_u = min\{\epsilon_1, \epsilon_2\}$ and require that $\epsilon \leq \epsilon_u$.

After a candidate controller $c(s)$ is designed, define $g(s) = p(s)c(s)$ such that the closed-loop system of Fig. 1 is equivalent to the unity-feedback closed-loop system of Fig. 2. Note that the nominal system is $g_0(s) := p_0(s)c(s)$, and the uncertainty is $\delta_g(s) := \delta(s)c(s)$. For robust stability of the system of Fig. 2, the following necessary conditions should be satisfied:

(B1) The nominal system $g_0(s) = p_0(s)c(s)$ is stable under unity-negative-feedback.

(B2) The family of uncertain systems $g(s)$ and its nominal system $g_0(s)$ have the same number of open-loop unstable poles.

If a candidate controller $c(s)$ has unstable zeros, one must ensure that there is no unstable pole-zero cancellation between the zeros of controller $c(s)$ and the poles of the family of $p(s)$ for all $\epsilon \leq \epsilon_u$. A similar analysis should be carried out for the possible unstable pole-zero cancellation between the unstable poles of controller $c(s)$ and the unstable zeros of the original systems $p(s)$. A candidate controller should be rejected if any forbidden pole-zero cancellation occurs.

The first problem posed in this paper is to determine the parametric stability margin ϵ_{max}, which is defined in [8] as the maximum ϵ for which the entire family of interval plants $p(s)$ is robustly stabilizable. The second problem is to synthesize a stabilizing controller $c_{op}(s)$ that can robustly stabilize the family of plants for all $\epsilon \leq \epsilon_{max}$.

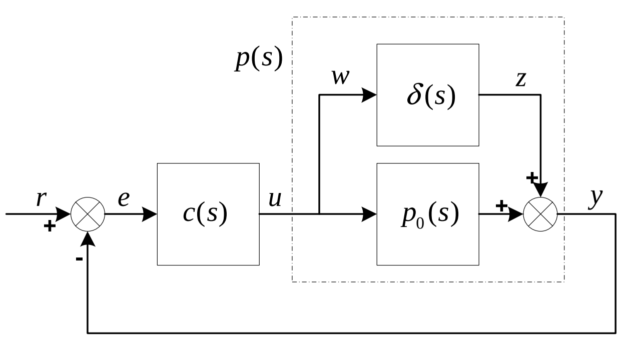

Figure 1: Negative feedback loop including the uncertain system $p(s) = p_0(s) + \delta(s)$ and a controller $c(s)$.

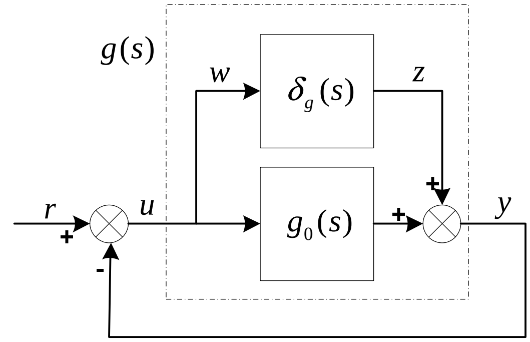

Figure 2: Stability analysis for the uncertain interval-system $g(s) = g_0(s) + \delta_g(s)$ under unity feedback.

3 Preliminaries

3.1 Robustness Analysis for Interval Systems: the Critical Direction Theory

Latchman *et al.* [9] [1] propose a necessary and sufficient frequency-domain robustness analysis technique, namely, the critical direction theory, which makes use of objects depicted in Figures 2 and 3. The solid curve in Fig. 3 describes the nominal system $g_0(j\omega)$. For a specific frequency point ω_i, the uncertainty family $g(j\omega_i)$ is characterized through the value-set $V(\omega_i)$. The following objects relevant to the subsequent discussion can be identified in Fig. 3:

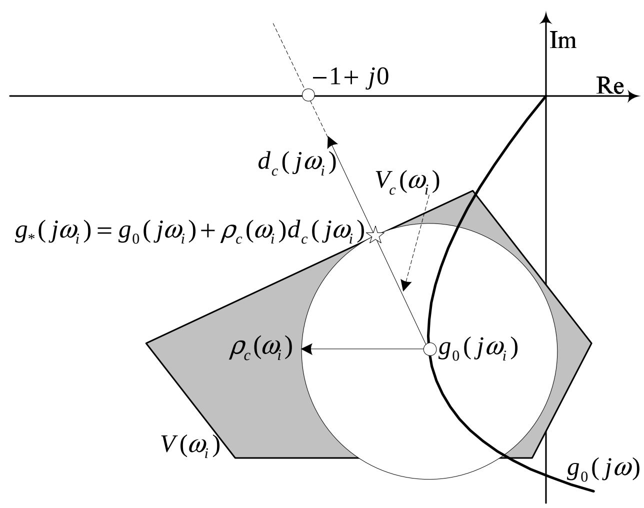

Figure 3: Illustration of entities used in the critical direction theory. The critical value set $V_c(j\omega_i)$ is in this case the straight-line segment joining $g_0(j\omega_i)$ and $g_*(j\omega_i)$.

1. The critical line is the directed line which originates at the nominal point $g_0(j\omega)$ and passes through the critical point $-1 + j0$.

2. The critical direction

$$d_c(j\omega) := -\frac{1+g_0(j\omega)}{|1+g_0(j\omega)|}$$

3. The value set

$$V(\omega) := \{g(j\omega) \ | \ g(j\omega) = g_0(j\omega) + \delta_g(j\omega)\}.$$

4. The critical value-set

$$V_c(\omega) \ := \ \{g(j\omega) \mid g(j\omega) = g_0(j\omega) + \alpha d_c(j\omega), \text{ for some } \alpha \in R_+\}.$$

5. The critical perturbation radius (CPR)

$$\rho_c(\omega) := \max_{\alpha \in R_+}\{\alpha \ | \ g_0(j\omega) + \alpha d_c(j\omega) \in V_c(\omega)\}. \quad (1)$$

6. The Nyquist robust stability margin

$$k_N := \left\| \frac{\rho_c(\omega)}{|1+g_0(j\omega)|} \right\|_\infty . \quad (2)$$

Theorem 1 *Consider the system $g(s)$ given in Fig. 2 which is nominally stable under unity feedback, and assume that $g(s)$ and $g_0(s)$ have the same number of open-loop unstable poles. Then, under the assumption that the critical value-set is convex, the uncertain system is stable under unity-negative-feedback if and only if $k_N < 1$.*

Proof: The proof is given in [9]. Note that only $V_c(j\omega)$ is required to be convex (*i.e., straight-line segment*), while $V(j\omega)$ can be highly nonconvex. Refer to [1] for the proof for the case of nonconvex critical value-sets. ■

The calculation of $\rho_c(\omega)$ is based on the description of the boundary of the value set [9]. The CPR's dependence upon the value set makes the results in this paper applicable to general perturbation structures such as multilinear, multiaffine and nonlinear perturbation [11]. In particular, for systems with affine parametric uncertainties, the CPR can be calculated via a systematic and numerically tractable algorithm since the boundary of the value-set consists of only circular arcs or line segments [1] [5].

3.2 Robustness Synthesis for Interval Systems: H_∞ Methods

As suggested in [2][8][14], the robustness synthesis problem formulated in Section 2 can be cast in the well-known mixed-sensitivity H_∞ synthesis framework. Using the standard notation shown in Fig. 4, the controller design procedure involves finding the minimizing solution to

$$\gamma = \min_{stabilizing\ c(s)} \|W_2(s)T_{ur}(s)\|_\infty, \quad (3)$$

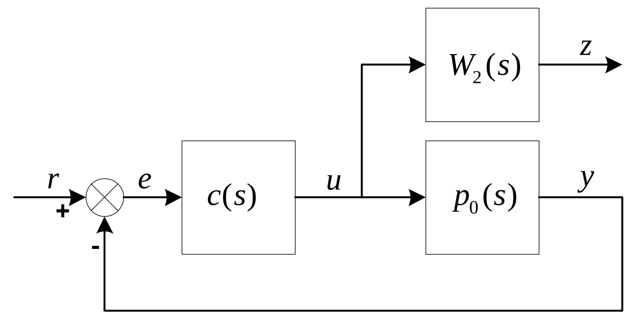

Figure 4: Mixed-sensitivity approach to the uncertain feedback system of Figure 1.

where $T_{ur}(s) := \frac{c(s)}{1+c(s)p_0(s)}$ is the controller sensitivity transfer function, and where the weight $W_2(s)$ must be selected to represent the plant perturbations in as exact a manner as possible. If $\gamma < 1$, then the minimizing solution $c(s)$ is a robustly stabilizing controller. Strictly speaking, the problem given in (3) is a simple (rather than mixed) sensitivity problem since the other sensitivity transfer functions $W_1(s)$ and $W_3(s)$ are zero [13] [12]. The choice of the weighting function $W_2(s)$ is critical in representing the *effective* part of the uncertainty $\delta(s)$.

In [8], the maximum perturbation radius (MPR) for the system $p(j\omega)$ at each frequency ω is defined as follows:

$$\rho_m(\omega) := \max_{\epsilon \in \Delta(\epsilon)} \ |\delta(j\omega)| = \max_{\epsilon \in \Delta(\epsilon)} \ |p(j\omega) - p_0(j\omega)|.$$

The constant worst-case uncertainty bound is just the largest value of the MPR over all frequencies, and is denoted by

$$\rho_m := \max_{\omega} \ \rho_m(\omega).$$

It is well-known that to guarantee the existence of a stabilizing controller in H_∞ synthesis, together with other requirements, the spectral radius of the product of the solutions of two Riccati equations associated with two Hamiltonian matrices should be less than one, *i.e.*, $\rho(X_\infty Y_\infty) < 1$ [13] [12]. To compute the parametric stability margin ϵ_{max}, the method proposed in [8] uses the constant MPR weighting, *i.e.*, $W_2(s) = \rho_m$, and then, ϵ and hence ρ_m is increased in small steps from an initial small value until an upper bound ρ_m^u is reached for which $\rho(X_\infty Y_\infty)$ approximates unity, *i.e.*, any further increase in ρ_m does not satisfy the spectral radius conditions anymore. Then, the stability margin ϵ_{max} corresponding to ρ_m^u can be found from a plot of ρ_m *vs.* ϵ.

4 Main Results

Recall that the critical direction theory reveals that only the uncertainty plants along the critical direction, not over the whole value set, need to be considered in solving for necessary and sufficient conditions for the robust stability of the system of Fig. 2. To analyze the conservatism that may be introduced by taking the constant uncertain bound based on the MPR as the weighting function $W_2(s)$, we now compare the value sets of system $p(s)$ and $g(s)$.

As indicated in Section 2, system $g(s)$ in Fig. 2 corresponds to $c(s)p(s)$ in Fig. 1. After multiplication by the controller $c(j\omega)$, the value set of $p(j\omega)$ may be shifted, rotated, and/or magnified or contracted to produce the value set of $g(j\omega)$. Fig. 5 illustrates the behavior of the corresponding variation caused by the controller $c(j\omega)$ between the value set of $p(j\omega_i)$ and that of $g(j\omega_i)$ at a frequency ω_i. Fig. 5 uses the following symbols: $g_*(j\omega_i)$ represents the intersection of the critical line originating from $g_0(j\omega_i)$ and the boundary of the value-set of $g(j\omega_i)$ as shown in Fig. 5 (b); and, $p_*(j\omega_i)$ denotes the intersection of the boundary of the value-set of $p(j\omega_i)$ and a certain line segment originating from $p_0(j\omega_i)$ as shown in Fig. 5 (a). This line segment is shifted and rotated to coincide with the critical line in the value-set of $g(j\omega_i)$. It is important to point out that it is the boundary point $p_\star(j\omega_i)$ that is critical for the robust stability analysis.

From the definition (1), it follows that

$$\begin{aligned} \rho_c(j\omega) &= |g_*(j\omega) - g_0(j\omega)| \\ &= |c(j\omega)p_*(j\omega) - c(j\omega)p_0(j\omega)| \\ &= |c(j\omega)||p_*(j\omega) - p_0(j\omega)| \end{aligned} \tag{4}$$

Hence,

$$\begin{aligned} \rho_c(j\omega) &\leq |c(j\omega)|\ \rho_m(\omega) \\ &\leq |c(j\omega)|\rho_m \end{aligned} \tag{5}$$

Clearly, conservatism is introduced by assigning $W_2(s) = \rho_m$, instead of $|W_2(j\omega)| = |p_\star(j\omega) - p_0(j\omega)|$.

4.1 The Constant ECPR Weighting Strategy

We now seek a non-conservative weight using the necessary and sufficient stability condition associated with the critical direction theory. Given a controller $c(s)$, we define the *effective* critical perturbation radius (ECPR) as

$$\rho_e(\omega) := \frac{\rho_c(\omega)}{|c_{op}(j\omega)|}, \tag{6}$$

where $\rho_c(\omega)$ is the CPR of the cascaded system $g(j\omega) = c_{op}(j\omega)p(j\omega)$ at frequency ω. Note that $\rho_c(\omega)$ is associated with the value-set of $g(j\omega)$, while $\rho_e(\omega)$ is associated with the value-set of $p(j\omega)$ as illustrated in Fig. 5. Now define the constant *effective* uncertainty bound as

$$\rho_e := \max_{\omega}\ \rho_e(\omega). \tag{7}$$

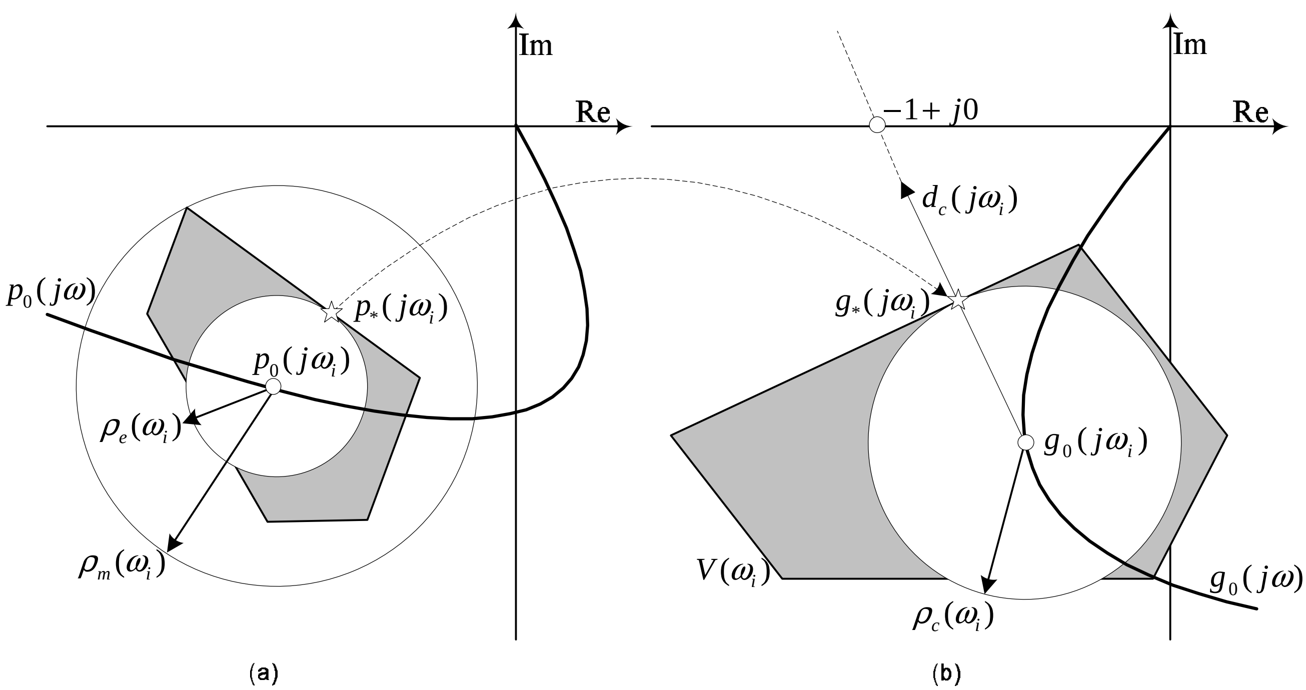

Figure 5: Complex maps between the original plant $p(j\omega)$ and the transformed system $g(j\omega) = c(j\omega)p(j\omega)$. The darkened areas represent the uncertainty value-sets. The point $g_*(j\omega_i)$ in (b) defines a disk of radius $\rho_c(\omega_i)$, whereas its image $p_*(j\omega_i)$ in (a) defines a disk of radius $\rho_e(\omega_i)$. The overbounding disk of radius $\rho_m(\omega_i)$ in (a) circumscribes the disk of radius $\rho_e(\omega_i)$, and leads to more conservative robust-stability estimates.

A controller $c_{op}(s)$ is considered optimal in terms of stability capacity if it minimizes $\|T_{zr}(s)\|_\infty$ in Fig. 4 among all stabilizing controllers and ensures that $\rho(X_\infty Y_\infty)$ is approximately unity.

Theorem 2 *For the* static *weighting approach where* $W_2(s)$ *is an constant, the* exact constant *ECPR weight is*

$$\rho_e \approx \frac{1}{\|T_{ur}(j\omega)\|_\infty}. \tag{8}$$

Proof: The stabilizing controller $c_{op}(s)$ achieves marginal stability at ϵ_{max}. This implies that

$$\rho(X_\infty Y_\infty) \approx 1,$$

Hence,

$$\|T_{zr}(s)\|_\infty \approx 1. \tag{9}$$

Note that $T_{zr}(s) = T_{ur}(s)W_2(s)$, and that after assigning $W_2(s) = \rho_e$, equation (9) yields

$$\|T_{ur}(j\omega)\rho_e\|_\infty = \rho_e\|T_{ur}(j\omega)\|_\infty \approx 1,$$

which is equivalent to (8). ■

Theorem 3 *At* ϵ_{max}*, the associated stabilizing controller* $c_{op}(s)$ *satisfies* $k_N \approx 1$ *and* $\|T_{zr}(s)\|_\infty \approx 1$.

Proof: These properties can be easily derived from Theorem 2. ■

Therefore, to keep the closed-loop system stable with controller $c(s)$, the constant effective uncertainty bound should satisfy

$$\rho_e \leq (\min_{stabilizing\ \ c(s)} \|T_{ur}(s)\|_\infty)^{-1}. \tag{10}$$

Following the definition of the constant effective uncertainty bound, the stability margin ϵ_{max}, corresponding to $(\min_{stabilizing\ \ c(s)} \|T_{ur}(s)\|_\infty)^{-1}$ using the H_∞ methods in conjunction with the constant weighting strategy, can be found from a plot of ρ_e *vs.* ϵ, instead of ρ_m *vs.* ϵ. Thus, by interpreting the static weight W_2 as a constant ECPR weight ($W_2(s) = \rho_e$), instead of a constant MPR weight, a larger parametric stability margin may be found.

5 Example

Consider the interval plant given in [2] [4] [6] [8],

$$p(s, \mathbf{q}) = \frac{5s + q_1}{s^2 + q_2 s + q_3}$$

with intervals

$$q_1 \in [4-\epsilon, 4+\epsilon], q_2 \in [2-\epsilon, 2+\epsilon], q_3 \in [-15-\epsilon, -15+\epsilon],$$

where ϵ represents the level of uncertainty. Note that to satisfy the assumption in Section 2, $\epsilon_u = 7.5$ [4] [8].

The mixed-sensitivity problem can be solved using state-space augmentation, as done in [2] [8]. Alternatively, it can be solved by constructing the transfer-function augmentation

$$p_0(s) = \frac{5s + 4}{s^2 + 2s - 15},$$

$$W_1(s) = W_3(s) = 0, \text{ and } W_2(s) = \beta \text{ where } \beta \in R_+.$$

Assign β an arbitrary initial value for which a stabilizing controller can be designed by H_∞ methods with that value as the weight, then increase β until an β_{max} is reached where $\rho(X_\infty Y_\infty)$ approximates unity. Using β_{max} as the constant weighting function $W_2(s)$, an optimal controller $c_{op}(s)$ under merely constant weighting approach can be found. Notice that this can be done efficiently through a bisection algorithm as done by the *Matlab Control Toolbox* using $\gamma - iteration$. It turns out that at $W_2 = 0.3950$, $\rho(X_\infty Y_\infty)$ approximates unity and the following controller is found

$$c(s) = \frac{3603.7935s + 18018.9673}{s^2 + 1434.5016s - 2312.4499}. \tag{11}$$

Hence $\beta = 0.3950$ is the limiting value of $W_2(s)$. Referring to the dashed line in Fig. 6 which reflects the variation of ρ_m *vs.* ϵ, it is observed that $\epsilon = 2.8300$ when $\rho_m = 0.3950$ (the limiting value of W_2). The value $\epsilon = 2.8300$ was proposed in [8] as the parametric stability margin in the case of the constant weighing method.

However, it turns out that this controller can stabilize uncertainties far larger than the level of 2.8300. There are many ways to check the stability margin of the interval plant in Fig. 1 with the controller given in (11). The first one is to check the stability of the 32 extremal segments of the value-set of $c(j\omega)p(j\omega)$ for each frequency ω while increasing the uncertainty ϵ until any one of the extremal segments passes through $-1 + j0$ at a certain frequency. The second method is based on Theorem 1 for system $g(s) = c(s)p(s)$, *i.e.*, calculating the Nyquist stability margin k_N while increasing ϵ until $k_N \approx 1$. The third approach, also the recommended one, is based on the linear programming algorithm posed in [2] and [10]. Using all three methods, the parametric stability margin is found to be $\epsilon_{max} = 5.2511$ (> 2.8300). Furthermore, it is straightforward to check that there is no unstable pole-zero cancellation between the unstable pole of $c_{op}(s)$ (11) and the zeros of $p(s)$ with uncertainty level ϵ up to ϵ_u.

Referring to the solid line in Fig. 6 which reflects the variation of ρ_e *vs.* ϵ, it is observed that when $\rho_e = 0.395$ (the limiting value of W_2), $\epsilon = 5.2511$ which is exactly the stability margin calculated for system of Fig. 1 with the H_∞ controller $c(s)$ (11). Therefore, the parametric stability margin calculated by the MPR weighting approach ($\epsilon = 2.8300$) is only about 54 percent of that obtained by the ECPR weighting approach ($\epsilon = 5.2511$). Furthermore, $\rho_e = 0.3950$ at $\epsilon_{max} = 5.2511$, *i.e*, the inverse of the minimum value of $\|T_{ur}(s)\|_\infty$, which is consistent with Theorem 2.

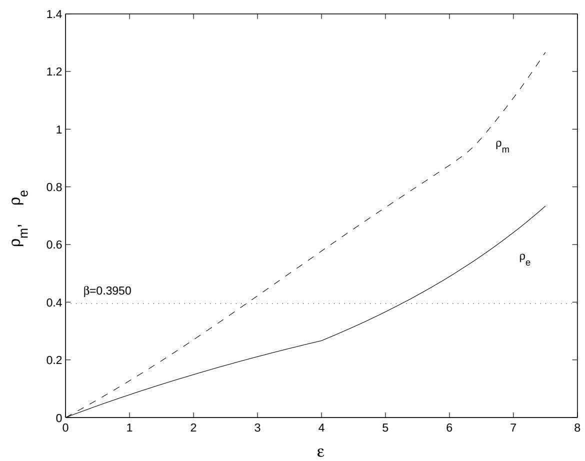

Figure 6: Plots of ρ_m and ρ_e radii as functions of the uncertainty-size parameter ϵ. The value $\beta = 0.3950$ is the limiting value of the static weight $W_2(s)$ found via a standard unstructured H_∞ approach.

Remark 1: Even though this paper shows that the ECPR weighting strategy leads to a much larger parametric stability margin than the MPR weighting method does, only constant weighting methods are considered. Referring to (5), the *exact* weighting function should be $p_*(s)-p_0(s)$, which means a dynamic weighting function should be used to further reduce conservatism in calculating the parametric stability margin. The use of *dynamic* weighting functions will be considered in a future paper.

Remark 2: It should be noted that the controller must be bi-proper (proper with proper inverse) if extremely large frequencies are considered since the magnitude of a strictly proper controller vanishes as ω goes to infinity, and in that case $\rho_e(\omega) = \rho_c(\omega)/|c(j\omega)|$ tends to infinity.

6 Conclusions

This paper focuses on the H_∞ methods with the constant weighting strategy for systems with parametric uncertainties. The conservatism of earlier work in the literature is analyzed and a much larger parametric stability margin is obtained using the critical direction theory. The analysis in this paper holds promise for determining even larger stability margins and associated robust controllers by exploring dynamic weighting strategies.

References

[1] Baab, C. T., J. C. Cockburn, H. A. Latchman, and O. D. Crisalle, "Generalization of the Nyquist Robust Stability Margin and its Application to Systems with Real Affine Parametric Uncertainties," *International Journal on Robust and Nonlinear Control*, vol. 11, 2001, pp. 1415-1434.

[2] Bhattacharyya, S. P., H. Chapellat, and L. H. Keel, *Robust Control: the Parametric Approach*, Prentice Hall, 1995.

[3] Cho, T. S., S. W. Choi, and Y. C. Kim, "Robust Design of PID Controller for Interval Plants," SICE, Morioka, 1999, pp. 933-936.

[4] Datta, K. B., and V. V. Patel, "H_∞-based Synthesis for a Robust Controller of Interval Plants," *Automatica*, vol. 32, No. 11, 1996, pp. 1575-1579.

[5] Fu, M., "Computing the frequency response of linear systems with parametric perturbation," *Sys. and Contr. Letters*, vol. 15, 1990, pp. 45-52.

[6] Henrion, D., and O. Bachelier, "Low-order Robust Controller Synthesis for Interval Plants," Technical Report No. 98469, LAAS-CNRS, Toulouse, France, 1998.

[7] Ho, M., A. Datta, and S. P. Bhattacharyya, "Design of P, PI and PID Controllers for Interval Plants," *Proceedings of the ACC*, Philadelphia, USA, 1998.

[8] Keel, L. H., S. Bhattacharya, and S. P. Bhattacharyya, "Robust Stabilizer Synthesis for Interval Plants using H_∞ Methods," In CDC, IEEE, 32nd, San Antonio, 1993, pp. 3003-3008.

[9] Latchman, H. A., O. D. Crisalle, and V. R. Basker, "The Nyquist Robust Stability Margin - A New Metric for Robust Stability of Uncertain Systems," *International Journal on Robust and Nonlinear Control*, vol. 7, 1997, pp. 211-226.

[10] Latchman, H. A., O. D. Crisalle, C. T. Baab, and B. Ji, "The Exact Calculation of Real Stability Radii of Interval System," SSST 32th, Alabama, USA, 2002.

[11] Polyak, B. T., and J. Kogan, "Necessary and Sufficient Conditions for Robust Stability of Linear System with Multiaffine Uncertainty Structure," *IEEE Trans. Auto. Control*, Vol. 40, 1995, pp. 1255-1260.

[12] Sánchez-Peña, R. S., and M. Sznaier, *Robust Systems – Theory and Applications*, John Wiley & Sons, Inc., 1998.

[13] Zhou, K., and J. C. Doyle, *Essentials of Robust Control*, Prentice Hall, 1998.

[14] *Mu Toolbox*, the Mathworks Inc.