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Robustness of the Smith Predictor with Respect to Uncertainty in the Time-Delay Parameter

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Abstract

The robust stability of the Smith predictor scheme for time-delay compensation is analyzed with respect to uncertainty in the time-delay parameter. Appropriate definitions are introduced for the phase margin and complementary phase margin of nonminimum-phase systems, and are used to characterize the domains of stability for the closed loop using a Nyquist approach. Rigorous arguments are used to show that the stability domain is discontinuous when the nominal Nyquist function has more than one magnitude-crossover points. Robust stability results are given for the cases where the Nyquist plot has no magnitude-crossover points, and when it has a single or multiple crossover points. The results are illustrated through an example.

1. Introduction

The problem of designing controllers for plants with time delay is particularly challenging because the onset of instability may be particularly difficult to predict. This complicates the tuning of the controller and may degrade the overall performance. The control scheme attributed to Smith [16] and widely known as the *Smith predictor* introduces a compensation loop inside the controller with the purpose of canceling the effect of the dead time on the dynamics of the closed loop. This facilitates the tuning of the controller and permits analyzing stability using conventional methods. Several modifications of the Smith predictor have been proposed over the years, including the introduction of feedforward features [13], improved compensation [7], and predictor-observer configurations [6]. A number of authors have extended the approach to multiple-input multiple-output systems [13] [4] [8] [1] [12].

Even though the Smith predictor is normally not recommended for use in disturbance rejection problems, it is a highly effective controller for set-point tracking applications when the dead-time of the plant is precisely known. Unfortunately, the performance of the controller can be very poor, or worse yet, the loop may become unstable, when the dead-time parameter of the plant model used for designing the Smith predictor is not equal to the dead time of the actual plant under control. This problem has been considered extensively by Palmor [13] who discusses the stability properties of the scheme when there is uncertainty in the time-delay parameter. Other robustness issues are addressed in [11].

Previous work has not addressed a surprising stability behavior of the Smith predictor observed for certain plants when there is time-delay mismatch. More specifically, consider a Smith predictor designed with a fixed nominal time-delay value that is not equal to the actual delay of the plant being controlled. Experience in analyzing these systems shows that in certain circumstances the loop may become unstable if the actual delay of the plant is slightly smaller than the nominal delay. Such behavior is surprising and defies intuition because in this case the Smith predictor is designed using an overestimated delay, a worst-case scenario that is often expected to ensure stability. A second challenge to the designer's intuition arises when the analysis shows that the loop is stable should the value of the plant delay be smaller yet. These observations indicate that the range of time-delay values for which a given Smith predictor design delivers a stable loop is not necessarily continuous. The objective of this work is to quantify the robustness of the Smith predictor design with respect to errors in the time-delay parameter.

2. Smith Predictor Design

Consider the plant

$$p(s,\theta) = h(s)e^{-\theta s} \tag{1}$$

and its associated nominal model

$$p_o(s) = h_o(s)e^{-\theta_o s} \tag{2}$$

where $h(s)$ and $h_o(s)$ are strictly proper rational transfer functions that respectively represent the undelayed part of the plant and of the model, and where $\theta \geq 0$ and $\theta_o \geq 0$ are respectively the time-delay parameters for the plant and for the model. It is assumed that while the undelayed part of the plant (1) is exactly known (*i.e.*, $h(s)=h_o(s)$) the value of the time-delay parameter θ is uncertain in the sense that it belongs to the set

$$Q:=\{\theta\in\Re : \theta\geq 0, \theta_o+\delta\theta^-\leq\theta\leq\theta_o+\delta\theta^+\} \\ =\left[\theta^-,\theta^+\right] \qquad (3)$$

where θ_o is a known nominal time-delay value, and $\delta\theta^-\leq 0$ and $\delta\theta^+\geq 0$ are known bounds on the time-delay error. The condition $\theta\geq 0$ (or its equivalent condition $\delta\theta^-\geq-\theta_o$) is included in (3) as a causality constraint. Hence the uncertainty description Q is a continuous real interval that contains the nominal delay θ_o, and that has extreme points θ^- and θ^+ which in turn are functions the known bounds $-\theta_o\leq\delta\theta^-\leq 0$ and $\delta\theta^+\geq 0$.

The classical Smith predictor architecture is shown in Figure 1, where the inner loop is a function of only the nominal plant model (2), and where $c(s)$ is an arbitrary linear controller. In order to keep the analysis concise, it is assumed that the nominal transfer function $h_o(s)$ is stable. Under these conditions the closed loop of Figure 1 is said to be stable if the following transfer functions are stable:

$$\frac{y(s)}{r(s)}=\frac{c(s)h(s)e^{-\theta s}}{1+c(s)h_o(s)(1-e^{-\theta_o s})+c(s)h(s)e^{-\theta s}} \qquad (4a)$$

$$\frac{u(s)}{r(s)}=\frac{c(s)}{1+c(s)h_o(s)(1-e^{-\theta_o s})+c(s)h(s)e^{-\theta s})} \qquad (4b)$$

It is assumed that the closed loop is stable for the nominal design case where $\theta=\theta_o$.

The Smith predictor is considered to be *robust* with respect to the time-delay uncertainty Q if the closed loop is stable for all time-delay parameters $\theta\in Q$.

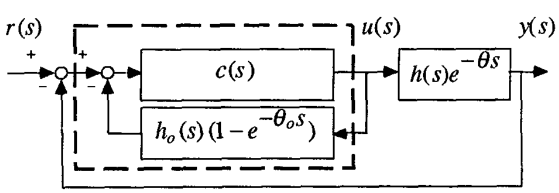

Figure 1. The Smith predictor architecture (area demarcated by dashed lines) for a plant with time delay.

3. Robust Stability Analysis

The stability analysis is carried out using the Nyquist function

$$L(s):=\frac{c(s)}{1+c(s)h_o(s)(1-e^{-\theta_o s})}h(s)e^{-\theta s} \qquad (5)$$

which is obtained from the operators of the closed-loop shown in Figure 1 using classical arguments [9]. The nominal Nyquist function is defined for the case when $\theta=\theta_o$ yielding

$$L_o(s):=\frac{c(s)}{1+c(s)h_o(s)(1-e^{-\theta_o s})}h_o(s)e^{-\theta_o s} \qquad (6)$$

For simplicity of notation, let

$$L_o(j\omega)=M_o(\omega)e^{-j\phi_o(\omega)} \qquad (7)$$

where $M_o(\omega)=|L_o(j\omega)|$ and $\phi_o(\omega)=\arg(L_o(j\omega))$ are respectively the magnitude and the phase of $L_o(j\omega)$ at a given frequency ω. Since for the case under consideration $h(s)=h_o(s)$, from (5) and (6) it follows that

$$L(j\omega)=L_o(j\omega)e^{-j\delta\theta\omega} \qquad (8)$$

where $\delta\theta=\theta-\theta_o$. It is important to remark that (8) implies that the net effect of a time-delay uncertainty $\delta\theta$ is to change the phase of the nominal transfer function $L_o(s)$ by an amount $-\delta\theta\,\omega$ while keeping the magnitude of $L_o(s)$ constant.

The analysis of stability on the Nyquist plane is challenging because (6) is a nonminimum-phase system which may feature multiple magnitude-crossover points. The analytical approach followed makes use of an appropriately defined phase margin that is suitable for use with systems displaying multiple crossover behavior. The task is facilitated with the assistance of the definitions given below.

Definition 1 (*Principal Phase*). *The principal phase of a nominal Nyquist point* $L_o(j\omega)=M_o(\omega)e^{-j\phi_o(\omega)}$ *is the unique polar-coordinate angle* $\hat{\phi}_o(\omega)\in(-\pi,\pi]$ *that corresponds to the Cartesian coordinates* $(\mathrm{Re}\,L_o(j\omega),\ \mathrm{Im}\,L_o(j\omega))$ *on the Nyquist plane.*

Definition 2 (*Phase Margin and Complementary Phase Margin*). *Let* ω_i^* *be a magnitude-crossover frequency of the nominal Nyquist plot* $L_o(j\omega)$. *Then the phase margin at this frequency is given by*

$$PM_i:=\begin{cases}\pi & \text{if } \hat{\phi}_o(\omega_i^*)=0\\ \hat{\phi}_o(\omega_i^*)-\pi\,\mathrm{sgn}(\hat{\phi}_o(\omega_i^*)) & \text{otherwise}\end{cases} \qquad (9)$$

and let the complementary phase margin be given by

$$\overline{PM}_i:=PM_i-2\pi\,\mathrm{sgn}(PM_i) \qquad (10)$$

where $PM_i\in(-\pi,\pi]$, $\overline{PM}_i\in(-2\pi,-\pi]\cup 0\cup(\pi,2\pi)$, *and where* sgn($x$) *is the signum function* (sgn(x):$= x/|x|, \forall x \neq 0$, *and* sgn($x$):$= 0$ *for* $x = 0$).

The key property of the phase margin and the complementary phase margin is that the following equalities hold at a magnitude-crossover frequency ω_i^*:

$$L_o(j\omega_i^*)e^{-jPM_i} = L_o(j\omega_i^*)e^{-j\overline{PM}_i} = 1 + j0 \qquad (11)$$

Note also that the phase-margin definition (9) is constructed such that it reduces to the classical definition for minimum-phase systems found in standard references [9] [15].

Definition 3 (*Stability Domain*). *The stability domain for the closed loop with the Smith predictor control scheme given in Figure 1 where there is uncertainty in the time-delay is the set*

$$S := \{\theta \in \Re : \theta \geq 0, \text{ and Eqs. (4a) and (4b) are stable}\} \qquad (12)$$

As mentioned in the introduction, the stability domain may not necessarily be a continuous interval.

Definition 4 (*Maximum Continuous Interval of Stability*). *The maximum continuous interval of stability* $\Theta_{\max}$ *is the largest continuous subset of the stability domain S that contains the nominal time delay* θ_o

The following corollary which follows directly from the preceding definitions establishes a relationship between the stability domain and the robustness of the Smith predictor.

Corollary 1. *The closed loop is stable for a given time delay* θ *if and only if* $\theta \in S$; *furthermore, the Smith predictor is a robustly stable controller with respect to the uncertainty Q if and only if* $Q \subset S$.

3.1. Case of no magnitude-crossover points

The robust stability analysis is particularly straightforward for the case where the nominal Nyquist function does not have any magnitude-crossover points. Note that if the nominal Nyquist function (6) has no crossover points, then its magnitude is less than unity at all frequencies.

Theorem 1. *Let the plant* (1) *have a delay* $\theta = \theta_o + \delta\theta \geq 0$ *where* θ_o *is the nominal delay and* $\delta\theta$ *is a real uncertainty. If the nominal Nyquist function satisfies* $|L_o(j\omega)| < 1$ *at all frequencies then the closed loop with the Smith predictor scheme given in Figure 1 is stable for all time-delay uncertainties satisfying* $\delta\theta \geq -\theta_o$.

Proof. First the condition $\delta\theta \geq -\theta_o$ is necessarily imposed to preserve causality. Then note that equation (8) implies that $|L(j\omega)| = |L_o(j\omega)|$ for all time-delay uncertainties $\delta\theta$; therefore, given that by hypothesis $|L_o(j\omega)| < 1 \ \forall \omega$ it follows that $|L(j\omega)| < 1 \ \forall \delta\theta$, and it is concluded that $L(j\omega)$ cannot encircle the point $-1+j0$. Invoking the Nyquist stability criterion [9] it follows that the closed loop is stable. *Q.E.D.*

Theorem 1 states that the Smith predictor cannot be destabilized by any causal time-delay parameter when $L_o(j\omega)$ is strictly bounded inside the unit circle. Under these conditions it follows that the stability domain is $S = [-\theta_o, \infty)$; hence $Q \subset S$, and it is readily concluded that the Smith predictor is robustly stable with respect to all time-delay parameters $\theta \in Q$.

3.2 Case of a single magnitude-crossover point

When the nominal Nyquist function (6) has only one magnitude-crossover point at a frequency ω^* the amount of uncertainty tolerable in the time-delay before the onset of instability is closely related to the phase margin and to the complementary phase margin at ω^*, as indicated in the following theorem.

Theorem 2 (*Single Crossover Theorem*). *Let the plant* (1) *have a delay* $\theta = \theta_o + \delta\theta \geq 0$ *where* θ_o *is the nominal delay and* $\delta\theta$ *is a real uncertainty. Also let the nominal Nyquist function* $L_o(j\omega)$ *have a single crossover point at frequency* ω^* *characterized with a phase margin PM and a complementary phase margin* $\overline{PM}$. *Then the closed loop with the Smith predictor scheme given in Figure 1 is stable if and only if*

$$\theta_{ext}^- < \theta < \theta_{ext}^+ \qquad (13)$$

where

$$\theta_{ext}^- = \theta_o + \delta\frac{PM}{\omega^*} + (1-\delta)\frac{\overline{PM}}{\omega^*} \qquad (14a)$$

$$\theta_{ext}^+ = \theta_o + \delta\frac{\overline{PM}}{\omega^*} + (1-\delta)\frac{PM}{\omega^*} \qquad (14b)$$

and

$$\delta = \delta(\hat{\phi}_o(\omega^*)) := \begin{cases} 0 & \text{if } \hat{\phi}_o(\omega^*) \in (-\pi, 0] \\ 1 & \text{if } \hat{\phi}_o(\omega^*) \in (0, \pi] \end{cases} \qquad (15)$$

Proof. Inequality (13) is demonstrated by proving that $S = (\theta_{ext}^-, \theta_{ext}^+)$. It suffices to show that the loop is unstable at the extreme points $\theta = \theta_{ext}^-$ and $\theta = \theta_{ext}^+$, and that it is stable for $\theta_{ext}^- < \theta < \theta_{ext}^+$. Consider the case where $\theta = \theta_{ext}^+$ with θ_{ext}^+ given by (14b) and analyze individually the two cases of principal phase angle appearing on the right side of (15). First, when $\hat{\phi}_o(\omega) \in (-\pi, 0]$, equation (15) yields $\delta = 0$ and (14b) reduces to $\theta_{ext}^+ = \theta_o + PM/\omega^*$, or equivalently, $\delta\theta = \theta_{ext}^+ - \theta_o = PM/\omega^*$. Using this expression for $\delta\theta$

in (8) with $\omega = \omega^*$ yields $L(j\omega^*) = L_o(j\omega^*)e^{-j\,PM}$, and finally, invoking (11) it follows that $L(j\omega^*) = 1 + j0$ and it is concluded that the loop is unstable. Second, when $\hat{\phi}_o(\omega) \in (0, \pi]$, following analogous arguments leads to $L(j\omega^*) = L_o(j\omega^*)e^{-j\overline{PM}_i} = 1+j0$ and to the conclusion that the loop is unstable. This proves that the loop is unstable when $\theta = \theta^+_{ext}$. The same reasoning can be applied to readily conclude that the loop is unstable when $\theta = \theta^-_{ext}$, hence proving that both end points of the stability domain S are unstable. The proof is concluded by recognizing that by the definitions of the phase margin and the complementary phase margin, the function $L(j\omega) = L_o(j\omega)e^{-j(\theta-\theta_o)\omega}$ cannot encircle the critical point -1+j0 in the frequency range $\theta^-_{ext} < \theta < \theta^+_{ext}$. *Q.E.D.*

Theorem 2 states that when the nominal Nyquist function has only one magnitude-crossover point the Smith predictor designed on the basis of the nominal parameter θ_o ensures that the closed loop is stable only for those values of θ that satisfy $\theta^-_{ext} < \theta < \theta^+_{ext}$. Equivalently, the stability domain for this case is the continuous interval $S = (\theta^-_{ext}, \theta^+_{ext})$. It can readily be concluded that the Smith predictor is robustly stable with respect to the uncertainty domain Q given in (3) if and only if $Q \subset S$, or equivalently, if and only if

$$(i)\;\; \theta_o + \delta\theta^- > \theta^-_{ext}$$

and

$$(ii)\;\; \theta_o + \delta\theta^+ < \theta^+_{ext}$$

Inequalities (i) and (ii) are necessary and sufficient conditions for the robust stability of the Smith predictor for the case where there is a single magnitude-crossover point in the nominal Nyquist function.

3.3 Case of multiple magnitude-crossover points

The case where the nominal Nyquist function (6) has multiple magnitude-crossover points introduces a discontinuity in the stability domain, as shown in the following lemma.

Lemma 1 (*Discontinuity Lemma*). *Let the plant* (1) *have a delay $\theta = \theta_o + \delta\theta \geq 0$ where θ_o is the nominal delay and $\delta\theta$ is a real uncertainty. Then if the nominal Nyquist function $L_o(j\omega)$ has n crossover points at frequencies ω_i^*, $i = 1, 2, \cdots, n$, $n>1$, the stability domain S is discontinuous.*

Proof. Consider two consecutive magnitude-crossover points of the nominal Nyquist plot at frequencies ω_1^* and ω_2^* such that $\omega_2^* > \omega_1^*$. Without loss of generality assume that $PM_2 > PM_1 > 0$ and that $|L_o(j\omega)| > 1$ at frequencies $\omega_1^* < \omega < \omega_2^*$. Let $\tilde{\theta}_1 = PM_1 / \omega_1^*$ and $\tilde{\theta}_2 = PM_2 / \omega_2^*$ be perturbations in the time delay. Clearly, $\tilde{\theta}_2\omega_2^* > \tilde{\theta}_1\omega_1^*$,

$$L(j\omega_1^*) = L_o(j\omega_1^*)e^{-\tilde{\theta}_1 j} = -1 + j0,$$

and

$$L(j\omega_2^*) = L_o(j\omega_2^*)e^{-\tilde{\theta}_2 j} = -1 + j0.$$

Now consider $L(j\omega) = L_0(j\omega)e^{-\tilde{\theta}_3 j}$ with $\tilde{\theta}_3 = \tilde{\theta}_2 + \varepsilon$, and select ε such that $0 < \varepsilon < (\tilde{\theta}_2\omega_2^* / \omega_1^*) - \tilde{\theta}_1$. For this latter Nyquist function the phase margin at $\omega = \omega_1^*$ is still positive, but the phase margin at $\omega = \omega_2^*$ is negative; therefore, the point -1+j0 is straddled by the crossover points at frequencies ω_1^* and ω_2^*. This in turn implies that the phase crossover frequency ω_{180} satisfies $\omega_1^* < \omega_{180} < \omega_2^*$. Given that $|L(j\omega)| > 1 \;\; \forall \omega \in (\omega_1^*, \omega_2^*)$ it follows that $|L(j\omega_{180})| > 1$ and it can be concluded that $L(j\omega)$ encircles the critical point -1+j0. *Q.E.D.*

Since S is discontinuous, then it is of interest to determine the maximum continuous subset of S that contains the nominal time-delay parameter θ_o. This information is crucial in assessing the robust stability of the Smith predictor with respect to the uncertainty description (3) which is obviously a continuous interval.

Theorem 3 (*Multiple Crossover Theorem*). *Let the plant* (1) *have a delay $\theta = \theta_o + \delta\theta \geq 0$ where θ_o is the nominal delay and $\delta\theta$ is a real uncertainty. Also let the nominal Nyquist function $L_o(j\omega)$ have multiple crossover points at frequencies ω_i^*, $i = 1, 2, \cdots, n$, $n>1$, with corresponding phase margins PM_i and complementary phase margins $\overline{PM}_i$. Then the maximum continuous interval of stability about the nominal delay θ_o is*

$$\Theta_{\max} = \left\{ \theta \in \Re : \theta \geq 0, \theta^-_{ext} < \theta < \theta^+_{ext} \right\} \tag{16}$$

where

$$\theta^-_{ext} = \max_{1 \leq i \leq n} \left\{ \theta^-_{ext,i} \right\} \tag{17a}$$

$$\theta^+_{ext} = \min_{1 \leq i \leq n} \left\{ \theta^+_{ext,i} \right\} \tag{17b}$$

and

$$\theta^-_{ext,i} := \theta_o + \delta_i \frac{PM_i}{\omega_i^*} + (1 - \delta_i)\frac{\overline{PM}_i}{\omega_i^*} \tag{18a}$$

$$\theta^+_{ext,i} := \theta_o + \delta_i \frac{\overline{PM}_i}{\omega_i^*} + (1 - \delta_i)\frac{PM_i}{\omega_i^*} \tag{18b}$$

and where $\delta_i = \delta(\hat{\phi}_o(\omega_i^*))$ is given by (15). *In order to satisfy the causality constraint $\theta > 0$ we adopt the convention that $\theta^-_{ext,i} := 0$ if equation* (18a) *yields $\theta^-_{ext,i} < 0$, and analogously, $\theta^+_{ext,i} := 0$ if equation* (18b) *yields $\theta^+_{ext,i} < 0$.*

Proof. For each crossover frequency ω_i^* it is possible to invoke Theorem 2 to find a continuous auxiliary interval $S_i = (\theta^-_{est,i}, \theta^+_{ext,i})$ with extreme points given by (18). Since by assumption the loop is stable for $\theta = \theta_o$, then each interval S_i contains a continuous subinterval where the loop is stable. It follows that $\Theta_{\max}$ must be the largest continuous subinterval of the running intersection $\cap_{i=1}^{n} S_i$ containing the nominal delay θ_o; therefore, it is concluded that $\Theta_{\max}$ is described by its extreme points θ^+_{ext} and θ^-_{ext} through inequality (16) and the optimizations (17). *Q.E.D.*

The previous results lead to the following robust stability corollary for the Smith predictor when there is uncertainty in the delay.

Corollary 2 (*Robust Stability*). *Let the plant* (1) *have a delay* $\theta = \theta_o + \delta\theta \geq 0$ *where* θ_o *is the nominal delay and* $\delta\theta$ *is a real uncertainty. Also let the nominal Nyquist function* $L_o(j\omega)$ *have multiple crossover points at frequencies* ω_i^*, $i = 1, 2, \cdots, n$, $n > 1$. *Then the Smith predictor scheme given in Figure 1 is robustly stable with respect to* Q *if and only if* $Q \subset \Theta_{\max}$, *or equivalently, if and only if*

$$(i)\ \ \theta_o + \delta\theta^- > \theta^-_{ext} \tag{19a}$$

and

$$(ii)\ \ \theta_o + \delta\theta^+ < \theta^+_{ext} \tag{19b}$$

where θ^-_{ext} and θ^+_{ext} *are defined in* (17a) *and* (17b).

Proof. First assume that the closed loop is robustly stable with respect to the continuous interval Q. This implies that the system is stable for all $\theta \in Q$. Since by definition $\Theta_{\max}$ is the largest continuous interval containing delays θ for which the system is stable, it is concluded that $Q \subset \Theta_{\max}$ because $\Theta_{\max}$ must contain all the elements of Q. Next, assume that $Q \subset \Theta_{\max}$. This implies that every element of Q is an element of $\Theta_{\max}$; hence, by the definition of membership into $\Theta_{\max}$ the closed loop is stable for every element of Q, *i.e.*, the system is robustly stable with respect to Q. Equations (19a) and (19b) follow after comparing the extreme points of the interval Q defined in (3) with those of $\Theta_{\max}$ defined in (16)-(17) and imposing the inclusion $Q \subset \Theta_{\max}$. *Q.E.D.*

4. Example

Consider the plant

$$p(s) = \frac{K}{\tau s + 1} e^{-\theta s}$$

where the parameters of the undelayed part of the plant are exactly known as $K = 1$ and $\tau = 1$, while the time delay is uncertain and of the form $\theta = \theta_o + \delta\theta$, where the nominal delay is $\theta_o = 1$. The controller $c(s)$ in the Smith predictor structure of Figure 1 is the proportional gain $c(s) = 4$. Finally, the uncertainty description for the time delay is

$$Q := \{\theta \in \Re : 0.8 \leq \theta \leq 1.5\} \tag{20}$$

For this example the nominal Nyquist function $L_o(j\omega) = 4\exp(-j\omega)/(j\omega + 1)$ has three magnitude-crossover points at the frequencies ω_1^*, ω_2^*, and ω_3^* shown in the second column of Table 1. The principal phase for each crossover point is reported in the third column of the table, followed by the phase margin and complementary phase margin calculated using the definitions (9) and (10). The value of the operator $\delta_i = \delta(\hat{\phi}_o(\omega_i^*))$ defined in Eq. (15) is also shown, along with the extreme delays $\theta^-_{ext,i}$ and $\theta^+_{ext,i}$ found for each crossover frequency via equations (18a) and (18b).

The extreme delays listed in the last two columns of Table 1 define the auxiliary intervals S_1 = (0, 2.8854), S_2 = (0.5668, 1.8206), and S_3 = (0.3462, 1.4425) associated with the crossover frequencies ω_1^*, ω_2^*, and ω_3^*, respectively. Applying equations (17)-(18) with $\theta_o = 1$ leads to

$$\theta^-_{ext} = \max\{0, 0.5668, 0.3462\} = 0.5668$$

$$\theta^+_{ext} = \min\{2.8854, 1.8206, 1.4425\} = 1.4425$$

Hence from (16)-(17) the maximum continuous interval of stability is $\Theta_{\max} = (0.5668, 1.4425)$. It is therefore concluded that the Smith predictor designed with $\theta_o = 1$ and $c(s) = 4$ designed in this example is not robustly stable with respect to the uncertainty set (20) because $Q \not\subset \Theta_{\max}$.

Figure 2 shows a stability diagram for the example. The stability domain $S = [0, 0.3462) \cup (0.5668, 1.4425) \cup (1,8206, 2.5320)$ is clearly discontinuous, as claimed in Lemma 1. This domain describes all the delay values that do not destabilize the loop. The diagram also shows that the closed loop is unstable for all delays satisfying $\theta \geq 2.5320$.

This example serves to highlight an important observation regarding the effect of time-delay uncertainty in Smith predictor design. Note that if the designer fixes $\theta_o = 1$ and then verifies that the loop is stable for the case where the actual plant has a larger delay, for example $\theta = 2$, it is incorrect to conclude that the closed loop will be stable for the case where the actual delay has values $\theta < 2$. This follows from the fact that the stability domain is discontinuous. In fact, from Figure 2 it is readily noted that the loop is unstable for $\theta = 1.5 < 2$.

5. Conclusions

A comprehensive analysis of the discontinuity of the stability domain of Smith predictor control loops has been carried out for the case where the time-delay parameter is uncertain. The results give necessary and sufficient conditions for the robust stability of the Smith predictor and are useful for the analysis of practical controllers.

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Table 1. Phase margins, complementary phase margins, and extreme plants for the system considered in Example 1.

i	ω_i^*	$\hat{\phi}_o(\omega_i^*)$	PM_i	$\overline{PM}_i$	$\delta(\hat{\phi}_o(\omega_i^*))$	$\theta^-_{ext,i}$	$\theta^+_{ext,i}$
1	0.7399	-1.7648	1.3768	-4.9064	0	0	2.8854
2	5.0122	0.9707	-2.1709	4.1123	1	0.5668	1.8206
3	5.7411	0.6055	2.5361	-3.7471	0	0.3462	1.4425

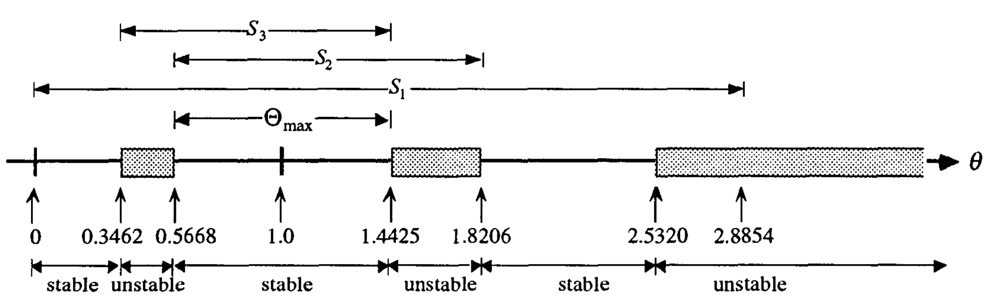

Figure 2. Stability diagram for the example with nominal time-delay $\theta_o = 1$. The shaded areas represent intervals of instability; hence, the stability domain is discontinuous. The maximum continuous interval of stability $\Theta_{max} = (0.5668, 1.4425)$ is obtained from knowledge of the end points of the auxiliary intervals S_i, i = 1, 2, 3.