Performance of Multi-level Modulation In MMSE Receiver Based CDMA Systems

Ali F. Almutairi
Haniph A. Latchman
Department of Electrical & Computer Engineering
University of Florida
Gainesville, Florida 32611-6130
Email: `almutair@ecel.ufl.edu`

Scott L. Miller
Department of Electrical Engineering
Texas A&M University
College Station, TX 77843-3128
Email: `smiller@ee.tamu.edu`

Abstract— **This paper studies the effect of using higher order modulation formats in the performance of minimum mean-squared error (MMSE) receiver based direct-sequence (DS) code-division multiple access (CDMA) systems in terms of bit error rate (BER) at different loading levels in additive white Gaussian noise (AWGN) and slow fading channels. The performance of BPSK, QPSK, and 16QAM modulation formats are compared and analytical and simulation results are presented in terms of the symbol error rates for these different modulation formats. A comparison of the rejection of the near-far effects for each modulation scheme is also presented. Under high loading level, 16QAM outperforms QPSK and BPSK for identical bandwidth and information rate while at a moderate loading levels, QPSK represents the best option.**

I. INTRODUCTION

Frequently, BPSK is often used as underlying modulation format with the MMSE receiver . The MMSE receiver, shown in Figure 1 which has been described in detail in [1], [2], [3], is a single user receiver which is known for its low complexity and ease of implementation.

Traditionally, higher level modulation has been used to achieve higher bandwidth efficiency (# of information bits transmitted in a given bandwidth). The price for the higher bandwidth efficiency is paid in terms of the required SINR to achieve the same error probability. In cellular systems,the main objective of the system designers is to increase the system capacity for a given quality of service and limited resources such as bandwidth.

To understand the advantages of the MMSE receiver, we need to describe briefly how it works. The received signal which consists of the desired user's signal, MAI, and Gaussian noise is fed at the chip rate into the equalizer until the N-tap delay line becomes full. After one symbol time, the equalizer content are correlated with the tap weights, **a**, and the result of this correlation is used to make a decision about which symbol was sent. These tap weights are updated every symbol interval to minimize the mean square error between the output of the filter and the desired output. In practice, the filter is trained for a reasonable period of time by a known training sequence to reach a tap weight vector that is close to the optimum weights. After the training period, the receiver switches to decision feedback mode. It has been shown in [4] that the decision directed mode proves to be troublesome in a fading channel. In deep fades, with the MMSE structure shown in Figure 1, incorrect decisions being fed back to the receiver cause the MMSE receiver to lose track of the desired signal. A modified MMSE receiver structure to overcome this problem was described in [4] for a BPSK modulation format. In [5], different tracking techniques of the desired user fading process and a general MMSE receiver structure for a fading channel were presented.

As noted in [1], if BPSK is used, the MMSE receiver becomes interference limited when the loading of the system becomes high enough and close to the processing gain. This threshold is reached because of the imperfect cancellation of the Multiple Access Interference (MAI) due to the lack of dimensions in the system.

One potential technique to increase the capacity of the MMSE receiver based CDMA system is through the use of higher order modulation schemes. The reasoning behind using multilevel modulation with the MMSE receiver can be presented as follows. Suppose an MMSE based CDMA system is operating using a modulation format such as QPSK. As the loading of the system increases, at some point, the MMSE receiver eventually becomes interference limited. This threshold in the number of users is reached because of imperfect suppression of the multiple access interference (MAI) due to the fact that the processing gain does not provide the MMSE receiver with enough dimensions to suppress all the interfering signals. Since the receiver now is in the interference-limited region, SINR can not be improved by simply increasing the transmitter power. This limitation can be overcome by increasing the processing gain (number of chips per symbols) by going to a higher order modulation format such as 16QAM. Doing so will move the MMSE receiver from the interference-limited region and will allow the receiver to suppress the multiuser interference and hence will increase the capacity of the CDMA system. Since the 16QAM modulation format needs a higher SINR to operate effectively, one can take advantage of the restored interference suppression capability of the MMSE receiver and increase the transmitted power to achieve a desirable performance.

0-7803-5538-5/99/$10.00

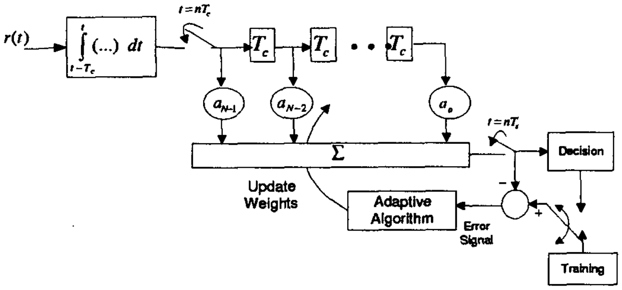

Fig. 1. The MMSE receiver structure

II. SYSTEM MODEL

In this section, an MMSE receiver based CDMA system will be described. There are K active users in the system. Each user is assumed to have a unique spreading waveform $c_j(t)$. It is assumed that all users transmit asynchronously over the same channel with carrier frequency w_o. The modulated signal of the jth user can be written as

$$\begin{aligned} s_j(t) &= \text{Re}\{\sqrt{2p_j}d_j(t)c_j(t)e^{jw_ot}\} \\ &= \text{Re}\{g_j(t)e^{jw_ot}\} \end{aligned} \tag{1}$$

where $g_j(t)$ is the complex envelope of $s_j(t)$, p_j is the transmitted power, and $d_j(t)$ is a complex baseband signalling format with symbol interval T_s. The waveform $c_j(t)$ is assumed to be in the polar form with chip interval T_c. Therefore, the processing gain N is equal to T_s/T_c. The bandpass received signal at the receiver is given by

$$r(t) = \text{Re}\{\sum_{j=1}^{K} \alpha_j(t)e^{j\theta_j(t)}g_j(t-\tau_j)e^{jw_ot}\} + n(t) \tag{2}$$

The variables τ_j, α_j, θ_j are the propagation delay, and the amplitude and phase of the fading process for the jth user. The process $n(t)$ is a real AWGN process with a spectral density of $N_o/2$. The fading amplitude is Rayleigh-distributed while the fading phase is uniformly-distributed.

It is assumed that the desired user is user 1 and the receiver has knowledge of its propagation delay. Without loss of generality, τ_1 can be set to zero. After converting the received signal to complex baseband, the signal is passed through a filter matched to the chip pulse shape. The matched filter has a scale factor of $\sqrt{2p_1T_c}$ associated with it. Based on these assumptions, The equalizer contents of the MMSE receiver are given by

$$\begin{aligned} \mathbf{r}(m) &= d_1(m)\alpha_1(m)e^{j\theta_1(m)}\mathbf{c}_1 \\ &+ \sum_{j=2}^{K} \sqrt{\frac{p_j}{p_1}}\alpha_j(m)e^{j\theta_j(m)} \Big[d_j(m)\tilde{\mathbf{f}}_j(l,\delta) \\ &+ d_j(m-1)\tilde{\mathbf{g}}_j(l,\delta)\Big] + \mathbf{n}(m) \end{aligned} \tag{3}$$

where the vector $\mathbf{n}(m)$ consists of elements of independent zero-mean complex Gaussian random variables whose real and imaginary parts have variances of $\frac{N}{(2E_s/N_o)}$ where E_s is the average energy per symbol. In the above equation, $\tau_j = l_jT_c + \delta_j$ where l_j is an integer and $0 \leq \delta_j < T_c$. The vectors $\tilde{\mathbf{f}}_j$ and $\tilde{\mathbf{g}}_j$ are defined as follows

$$\tilde{\mathbf{f}}_j(l,\delta) = \frac{\delta}{T_c}\mathbf{f}_j(N-l-1) + \left(1 - \frac{\delta}{T_c}\right)\mathbf{f}_j(N-l)$$

$$\tilde{\mathbf{g}}_j(l,\delta) = \frac{\delta}{T_c}\mathbf{g}_j(N-l-1) + \left(1 - \frac{\delta}{T_c}\right)\mathbf{g}_j(N-l)$$

where

$$\mathbf{f}_j(l) = (\mathbf{c}_j^{(l)} + \hat{\mathbf{c}}_j^{(l)})/2$$

$$\mathbf{g}_j(l) = (\mathbf{c}_j^{(l)} - \hat{\mathbf{c}}_j^{(l)})/2$$

$$\mathbf{c}_j^{(l)} = (c_{j,N-l}, c_{j,N-l+1}, \ldots, c_{j,N-1}, c_{j,0}, c_{j,1}, \ldots, c_{j,N-l-1})^T$$

$$\begin{aligned} \hat{\mathbf{c}}_j^{(l)} = (&-c_{j,N-l}, -c_{j,N-l+1}, \\ &\ldots, -c_{j,N-1}, -c_{j,0}, c_{j,1}, \ldots, c_{j,N-l-1})^T \end{aligned}$$

The variable $\mathbf{r}(m)$ can be written in the form

$$\mathbf{r}(m) = d_1(m)\alpha_1(m)e^{j\theta_1(m)}\mathbf{c}_1 + \tilde{\mathbf{r}}(m) \tag{4}$$

We define the following terms to be used in the next section. The autocorrelation matrix, $\mathbf{R}$, of the equalizer contents is defined as $\mathbf{R} = E\left[\mathbf{r}(m)\mathbf{r}(m)^H\right]$, The autocorrelation matrix of the MAI and noise part of the equalizer contents is given as $\tilde{\mathbf{R}} = E\left[\tilde{\mathbf{r}}(m)\tilde{\mathbf{r}}(m)^H\right]$ and the correlation between the desired user response and the received signal is given by $\mathbf{P}_i = E\left[d_i^*(m)\mathbf{r}(m)\right]$.

III. PERFORMANCE ANALYSIS

Despite the great deal of attention which has been given to the MMSE receiver in recent years, there has been very little detailed study of its performance with higher order modulation schemes. In this section, we present symbol error rates for the MMSE receiver using BPSK, QPSK, and 16QAM modulation schemes in AWGN and fading channels. For an AWGN channel let

$$\begin{aligned} \beta &= \mathbf{P}^H\tilde{\mathbf{R}}^{-1}\mathbf{P} && (5) \\ &= \frac{1-J_{min}}{J_{min}} && (6) \end{aligned}$$

where J_{min} is the minimum mean-squared error. A general form of the average symbol error can be written as

$$\hat{p} \approx BQ(\sqrt{A\beta}) \tag{7}$$

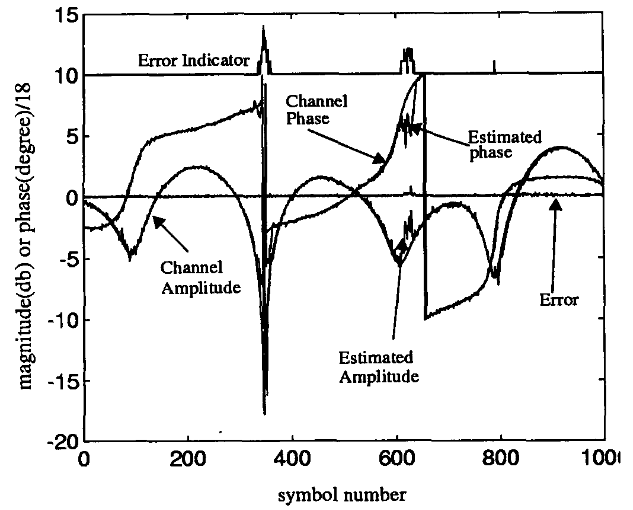

Fig. 2. Channel tracking using pilot symbols and linear prediction for 3 users using 16QAM modulation.

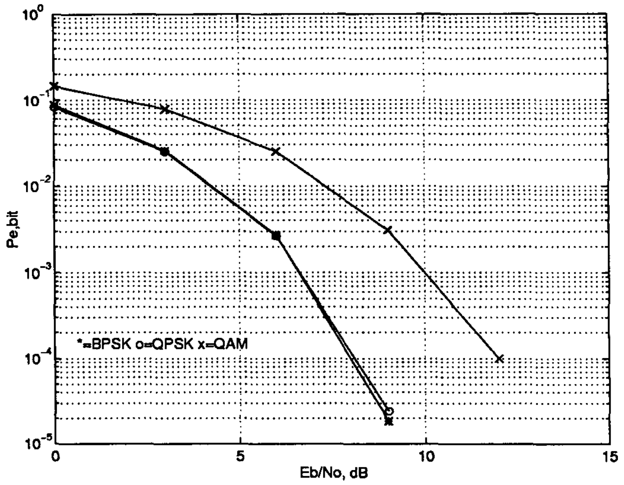

Fig. 3. The performance of BPSK, QPSK, and 16QAM in a Gaussian channel with 1 user.

for BPSK, $A = 2$ and $B = 1$; for QPSK, $A = 1$ and $B = 2$. For 16QAM, the average symbol error is given by

$$p_{e16QAM} = 3\hat{p}\left[1 - \frac{3}{4}\hat{p}\right] \tag{8}$$

where $A = \frac{1}{5}$ and $B = 1$.

For a fading channel, we will use the system model and the MMSE receiver structure presented in [5]. A frequency nonselective slow fading channel model is used. The tracking of the fading process amplitude and phase is accomplished through the use of periodically spaced pilot symbols and linear prediction as shown in Figure 2. In this case, channel estimates are made by feeding a linear prediction of previous channel estimates, some of which are based on pilot symbols. The phase and the amplitude estimates of the fading process from of the linear predictor are then used to remove the phase of the desired user fading from the input of the modified MMSE receiver and to scale the decisions from the modified MMSE receiver, respectively. To find the average symbol error, let

$$\gamma = \mathbf{c}_1^H \tilde{\mathbf{R}}^{-1} \mathbf{c}_1 \tag{9}$$

where the vector $\mathbf{c}_1$ represents the desired user's spreading sequence. A general form of the average symbol error can be written as

$$\hat{p} \approx F\left(1 - \sqrt{\frac{\gamma}{G+\gamma}}\right); \tag{10}$$

For BPSK, $F = \frac{1}{2}$ and $G = 2$; for QPSK,$F = 1$ and $G = 4$. For 16QAM, the average symbol error is obtained by substituting eqn. (10) into eqn. (8) for $F = \frac{1}{2}$ and $G = 20$.

For the single user case, it is easy to show that these results reduce to the well known results shown in the digital communications literature [7] and [8].

To obtain these results, we have used the Gaussian approximation for the output of the filter due to interference and noise. The justification for the Gaussian approximation is based on the central limit theorem by noting that the output of the filter is a sum of random variables with different probability density functions (pdfs). Therefore, the sum of these random variables at the output of the filter can be considered a Gaussian random variable. This approximation is widely used in evaluating conventional receivers [9]. Moreover this approximation is more accurate with the MMSE receiver since we have less interference at the output of the filter and more Gaussian noise [3]. Poor and Verdu in [10] have studied the behavior of the output of the MMSE receiver and found that the output is approximately Gaussian in many cases.

IV. SIMULATION RESULTS

In order to illustrate the improvements in the performance of the system in terms of BER and capacity, Figures 3, 4, and 5 show the performance of the MMSE receiver with BPSK, QPSK, 16QAM in a Gaussian channel for 1-, 20-, and 50- user CDMA systems. In these simulation results, the LMS algorithm step size used was $\mu = 0.1/TIP$ where TIP is the total input power and the received powers have a lognormal distribution with zero mean and 1.5 dB standard deviation. The processing gains are 31, 62, 124 for BPSK, QPSK, and 16QAM respectively. These values of the processing gains are chosen in order to have the bandwidths and the information rates equal for the three systems.

For a single user system, the bit error rate is the same for BPSK and QPSK and lower than that of 16QAM for a given Eb/No. When the load of the system increases to 20 users, the QPSK-based CDMA systems outperforms the BPSK-based system. The rate of improvement is faster for QPSK than for BPSK as the Eb/No increases. On the other hand, the 16

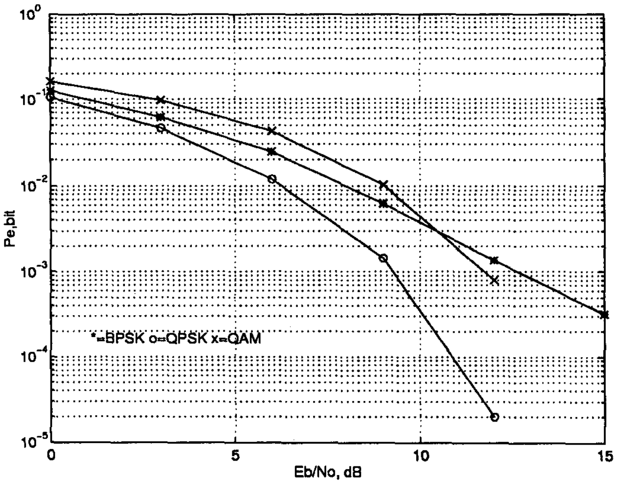

Fig. 4. The performance of BPSK, QPSK, and 16QAM in a Gaussian channel with 20 users.

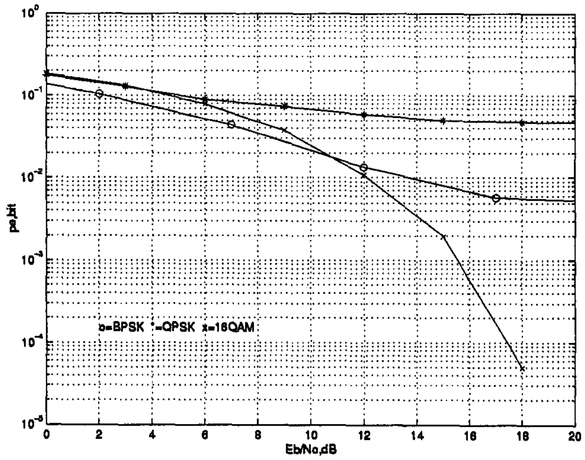

Fig. 5. The performance of BPSK, QPSK, and 16QAM in a Gaussian channel with 50 users.

QAM system starts about 1 dB worse than BPSK but at about $Eb/No = 11dB$ the 16QAM BER becomes lower than that of BPSK for a given Eb/No. With the load further increased to 50 users, both BPSK and QPSK will reach a point at which the bit error rate will become invariant to the increase in Eb/No. That basically means we can increase the load of the system by increasing the length of the processing gain without increasing the bandwidth or information rate by simply going to a higher order modulation.

Figure 6 shows how systems with different modulation formats deal with the near-far problem. In this case, the standard deviation σ_p (dB) of the interfering signal received power is varied while Eb/No is fixed at 5 dB for a load of 30 users. It is clear from the figure that, at this load, The MMSE receiver

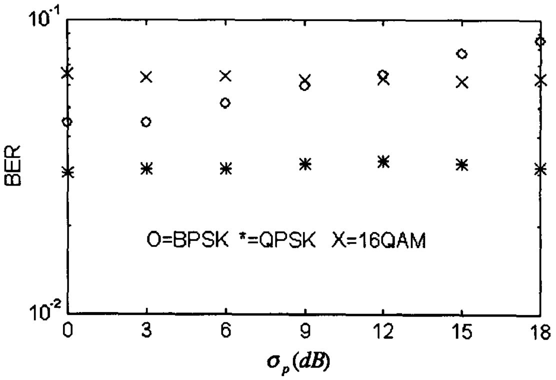

Fig. 6. BER of BPSK, QPSK, and 16QAM as a function of near-far ratio for 30 users.

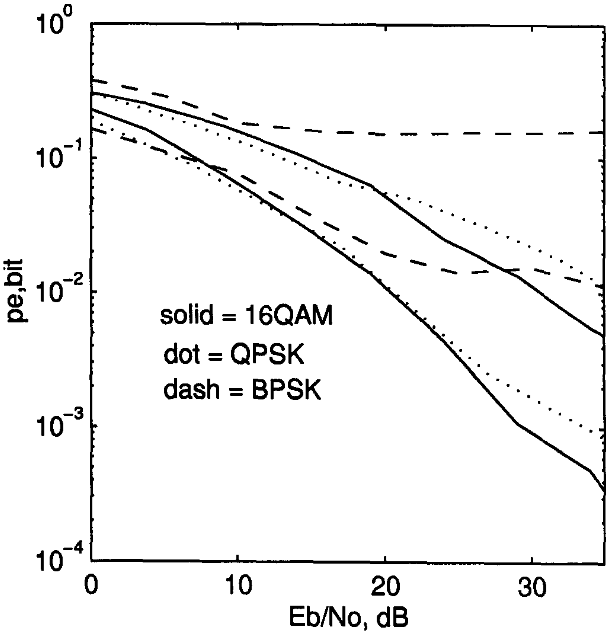

Fig. 7. Theoretical (bottom) and adaptive (top) BER of BPSK, QPSK, and 16QAM in a Rayleigh fading channel with 30 users

with the BPSK modulation format is not near-far resistant anymore. The QPSK and 16QAM based MMSE receiver systems are operating as a near-far resistant. Clearly, at this level of loading, one should choose a higher order modulation format to restore the near-far resistance of the MMSE reviver. If the system loading increased to a higher level, one should expect, the QPSK based system to lose its near-far resistant property.

Figure 7 shows the performance of the MMSE receiver based CDMA system using different modulation formats in a Rayleigh fading channel. For this environment, the Mobile speed was 5 mph, the frequency band was 900 MHz, the bit rate was 9600 bps, and the number of users was 30. In this figure the top 3 plots represent the adaptive, LMS, implementation of the MMSE receiver while the bottom 3 plots represent the theoretical results which were obtained by using eqn. (10). In the adaptive implementation, the phase and amplitude of the desired user's fading process were estimated using pilot symbols and linear prediction. The pilot symbol is sent every $10th$ symbol. As can be seen from the figure, with a BPSK modulation, the BER of the system does not improve when Eb/No increases. Performance of the system can be improved by using a higher order modulation. In addition, it can be seen that there is a loss of about one order of magnitude in terms of BER due to the adaptive implementation of the MMSE receiver.

V. CONCLUSION

In this paper, we have studied the performance of the MMSE receiver with BPSK, QPSK, and 16QAM modulation formats. We have shown that if we keep the bandwidth and information rate the same for BPSK, QPSK, and 16QAM, the 16QAM-based system outperforms the other modulation formats based system when the loading of the system is high. This performance improvement is made possible by increasing the processing gain and hence increasing the ability of the MMSE receiver to suppress the multiple access interference. Since the MMSE receiver will be operating in a near-far resistant region, the SINR can be increased to get acceptable performance of the 16QAM-based system. As we have seen, for highly loaded system, the system has an error floor in the case of BPSK and QPSK that is invariant to the increase of SINR. This performance limitation can be overcome by choosing a higher order modulation.

The performance of the adaptive form of the MMSE in a fading channel can be further improved if a more accurate estimate of the desired user fading process can be achieved.

REFERENCES

[1] Predrag B. Rapajic and Branka S. Vucetic. Adaptive receiver structures for asynchronous CDMA systems. *IEEE Journal on Selected Areas in Communications*, 12(4):685–697, May 1994.

[2] Upamanyu Madhow and Michael L. Honig. MMSE interference suppression for direct-sequence spread-spectrum CDMA. *IEEE Transactions on Communications*, 42(12):3178–3188, December 1994.

[3] Scott L. Miller. An adaptive direct-sequence code-division multiple-access receiver for multiuser interference rejection. *IEEE Transactions on Communications*, 43(2/3/4):1746–1755, February/March/April 1995.

[4] Afonso N. Barbosa and Scott L. Miller, *Adaptive detection of DS/CDMA signals in fading channels*, IEEE Transactions on Communications **46** (1998), no. 1, 115–124.

[5] Ali F Almutairi Scott L. Miller and Haniph A. Latchman. Tracking of multilevel modulation formats for DS/CDMA system in a slowly fading channel. *DIMACS Series in Discrete Mathimatics and Theoretical Computer Science*, page xxx, March 99.

[6] Haykin.S. *Adaptive Filter Theory*. Printice Hall, 1996.

[7] Israel Korn. *Digital Communications*. Van Nostrand Rienhold Company Inc, 1985.

[8] Joan G. Proakis. *Digital Communications*. McGraw-Hill, 1995.

[9] Michael B. Pursley. Performance evaluation for phase-coded spread-spectrum multiple-access communication - Part I: System analysis. *IEEE Transactions on Communications*, 25(8):795–799, August 1977.

[10] H.V. Poor and S. Verdu. Probability of error in MSSE multiuser detection. *IEEE Transactions in Information Theory*, 43(3):858–871, May 1997.