SOLID STATE DEVICE FOR THE MEASUREMENT OF LIGHT AIRCRAFT ROLL ATTITUDE

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Abstract— In this paper we describe a non-gyroscopic solid state device which provides an indication of the roll attitude of a light aircraft. The difference in altitude between the left and right wing tips is measured by commercially available solid state pressure sensors. For an average light aircraft with a wingspan of about 40 feet, the electrical signal corresponding to 1 degree in the roll attitude is of the order of 0.2 mV, but this measurement is also subject to ripple and drift noises of the same order of magnitude as the useful data. The paper describes a microprocessor-based digital filter which cancels these effects and produces a reliable indication of roll attitude. The filtering philosophy uses an observation technique based on the dynamics of the lateral motion of the aircraft system. It is shown that even an approximate knowledge of the mathematical model results in effective filter operation and acceptable instrument fidelity.

1 Introduction

Although the current state of the art in aviation technology is quite advanced, high technology devices are not generally made available to general aviation aircraft because of the absence of applied research in this area. In addition, mechanical and even the more recent optical gyroscopic devices for heading, roll and pitch measurements are prohibitively expensive. The absence of reliable and accurate flight data can result in catastrophic disaster especially when the purely human sensory interface of the pilot is impaired by night-time or other low-visibility flying conditions. The results presented in this paper are based on the premise that currently available system theoretical results may be invoked to produce significant improvements in flight data indicators for general aviation applications. In addition, the computational power now available from the fairly low-cost microprocessors enable the associated algorithms to be implemented on line in real-time.

The overall motivation in the work reported in this paper is to design a navigational device which will provide the pilot with an immediate, accurate, and reliable indication of the relevant flight conditions, and to present this information in the most convenient and familiar manner. The device, which should be available at a cost affordable for general aviation applications, is envisaged as a single visual display device with analog and digital flight data indications, as shown in Figure 1.

The instrument should include:

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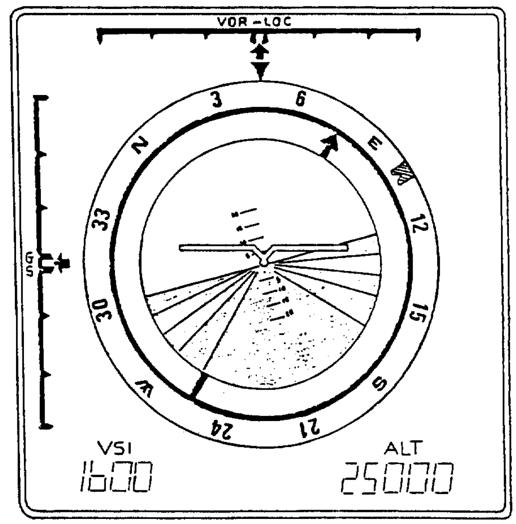

Figure 1: Proposed Integrated Navigational Instrument

1. A graphical display of the roll and pitch attitudes, deliberately designed to mimic the graphical presentation of conventional artificial horizons.
2. A graphical replication of the magnetic heading. This indication would take the form of a magnetic card displayed along the circumference of the circular display window.
3. A digital indicator of vertical speed, altitude, and other data could be provided as insets with the graphical data.

Clearly this integrated navigational instrument could replace several other conventional instruments on a typical aircraft display panel. Implementing such a system without the use of expensive gyroscopic devices is quite an ambitious task. In the present paper, we address only the question of obtaining the roll attitude angle using solid-state altitude sensors in conjunction with observer theory.

The remainder of the paper is organized as follows. Section 2 presents a simplifed linear model of the lateral motion of a typical light aircraft, and Section 3 evaluates the model to quantify the effects of parametric uncertainties and neglected non-linear effects. Section 4 considers 2 independent methods for computing the roll attitude of the aircraft, namely a differential pressure method and an observer-based method. Both of these methods are shown to yield error-prone estimates for the roll attitude. Section 5 introduces a novel microprocessor based filtering scheme to generate a refined roll measurement. Section 6 contains some brief concluding remarks and directions of ongoing and further research.

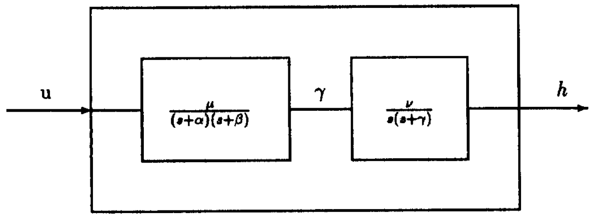

Figure 2 Roll Attitude Input-Output System

2 Mathematical Model

The lateral motion of the aircraft is approximated by the linearized model shown in the block diagram of Figure 2.
The input to the roll system is the aileron deflection u. u will be normalized to the range $-1 < u < 1$ with $u = 1(-1)$ representing the situation when the control wheel is all the way to the right (left). r, the roll attitude in degrees, is the quantity to be estimated and indicated as descibed in the rest of the paper, and h is the measured heading of the aircraft in degrees $0 < h < 360°$. For a typical light aircraft with ramp weight of about 4000 pounds and a 200 mph cruising speed, the model parameters $\alpha, \beta, \gamma, \mu$, and ν may be specified to within an order of magnitude, as follows:

$$\begin{array}{ccc} \alpha = 3s^{-1} & \beta = 0.1s^{-1} & \gamma = 0.15s^{-1} \\ \mu = 120°s^{-2} & & \nu = 1s^{-2} \end{array} \quad (1)$$

If we let

$$x_1 = r, \; x_2 = \dot{x}_1, \; x_3 = h, \; x_4 = \dot{x}_3, \quad x \in \mathcal{R}^4 \quad (2)$$

a state-space representation of the roll system is given by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.3 & -3.1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -0.15 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 120 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

3 Modeling Errors: Parameter Uncertainties and Nonlinearities

The simple mathematical model described above provides an approximate representation of the roll system of the aircraft. The main causes of divergence between this model and the real system are:

(a) System nonlinearites which have been ignored in deriving a linear model.

(b) Parametric uncertainties in the values of $\alpha, \beta, \gamma, \mu$, and ν which are known only to within an order of magnitude.

Despite these inaccuracies, however, we argue that this simple model correctly reproduces the fundamental property inherent

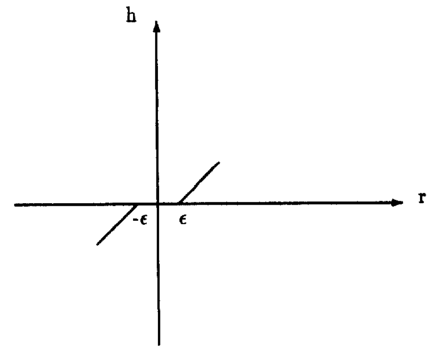

Figure 3: Dead-zone Nonlinearity

to the physics of the lateral motion of the aircraft, namely that zero roll implies and is implied by no change in heading, *i.e.*

$$\dot{h} \leftrightarrows r = 0 \quad (5)$$

It turns out that this is the basic information needed to pilot the aircraft in conditions of low visibility requiring the use of Instrument Flight Rule (IFR) operations.

Since the model is to be used for real flight scenarios in which $|r| < 30°$, the most important nonlinearity is the insensitivity *i.e.* the *dead zone*, in which the heading remains unchanged with r varying in the range $|r| < \epsilon°$ (see Figure 3). From the point of view of aircraft pilot, this nonlinearlity is accommodated by treating all $|r| < \epsilon°$ as $r = 0$ since the heading remains unchanged, *i.e.* $\dot{h} = 0$.

In the case of the parametric uncertainties of up to an order of magnitude in deviation away from nominal values of $\alpha, \beta, \gamma, \mu$, and ν, we acknowledge that a major question now becomes that of the robustness of the stability property in the presence of these uncertainties. Several methods are now available to evaluate the stability robustness for models of this kind with respect to various parametric uncertainties [3]. While these methods will not be discussed in the present paper, any of them can be used, with varying degrees of conservatism to ensure that the system is robustly stable. Under these conditions, despite the parametric uncertainties, the estimated roll indication can be used for IFR aircraft manoeuvres such as the following scenario: In order to steer the aircraft 40° to the right, the pilot will put the plane in a steady right bank of say 20° and will watch the heading variation until it reaches the desired value, at which time the pilot will return the aircraft to the horizontal attitude, $r = 0$.

Clearly the overall system will perform acceptably well despite the simplifying linearization and the parametric uncertainties, precisely because the controller in this case is essentially an expert feedback system, namely a trained human pilot. Thus the roll attitude indicator will serve as a useful aid to the pilot, and its effectiveness in this setting stems from the fact that it is used in actual navigation only indirectly as one input to the expert controller. While even an imprecise indication of roll may be useful in IFR conditions, it is very desirable that such an attitude indication, which the pilot uses to verify his degree of banking into a turn, should be as close as possible to the actual roll. An asymptotic observer to accomplish this goal is described in the next section.

4 Roll Attitude: Direct Measurement and Observer-based Estimate

We assume that the available measurements for heading, h, and aileron deflection u, are noiseless so that the objective is to provide the pilot with roll attitude information, for which only a crude and noisy measure is available, using the pressure sensors mounted on the wing tips. Since these pressure sensors are plagued by quite significant ripple and drift noises, the roll indication obtained directly from them would be quite unreliable. As an alternative, the roll attitude could be estimated using an observer, and the measured and estimated (observed) roll can then be combined to produce a refined roll indicator. Let us first consider the state observer; the differential pressure roll measurement is described later.

4.1 Asymptotic Roll Attitude Observer

The state of the system is completely observable [2] since

$$\text{Im}\left[C^T, A^TC^T, A^{T^2}C^T, A^{T^3}C^T\right] = \Re^4$$

$$C^T = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \end{vmatrix};\ A^TC^T = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{vmatrix};\ A^{T^2}C^T = \begin{vmatrix} \nu \\ 0 \\ 0 \\ -\gamma \end{vmatrix};\ A^{T^3}C^T = \begin{vmatrix} -\nu\gamma \\ \nu \\ 0 \\ \gamma^2 \end{vmatrix} \quad (6)$$

Thus a stable asymptotic observer can be designed and its state $z = \begin{vmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{vmatrix}$ will converge to the system state $x = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix}$. The observer equation is

$$\dot{z} = [A + KC]z + Bu - Ky$$

where

$$K = \begin{vmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{vmatrix};\ KC = \begin{bmatrix} 0 & 0 & K_1 & 0 \\ 0 & 0 & K_2 & 0 \\ 0 & 0 & K_4 & 0 \end{bmatrix}.$$

All the eigenvalues of the matrix $[A + KC]$ can be arbitrarily placed well to the left of the the imaginary axis by a proper choice of K_1, K_2, K_3, K_4. Thus, after some transient period, the state of the observer will asymptotically track the state of the system (A, B, C) with zero error, yielding, in the ideal case, the exact value of the roll attitude $r = x_1 = z_1$. Unfortunately the parameters of the system are uncertain and are as given in the nominal model (A, B, C) only to within an order of magnitude. Consequently in practice, the tracking error between the observer and actual system state will not vanish and, in particular $z_1 \neq x_1 = r$, namely the observer state z_1, will not coincide with the actual roll of the aircraft.

4.2 Differential Pressure Measurement of Roll Attitude

The difference in barametric pressure between the right and left wing tips is measured by using commercially available pressure sensors. Figure 4 shows the resulting voltage after amplification and high frequency noise filtering.

The resolution shown for these off-the-shelf pressure sensors is, in fact, very good; fractions of one foot can be clearly detected. This is certainly adequate resolution for a rough indication of roll attitude, since a 0.8 foot difference in altitude between the left and right wing tips corresponds to a roll of about 1° for an aircraft with a 40 ft wingspan. However, as indicated in Figure 4, the output of the pressure sensors is effected by a slow varying drift value which is of the same order of magnitude as the roll signal. Let us denote the output from the differential pressure sensors as $r + d$, where r is the true roll angle and d is the drift value.

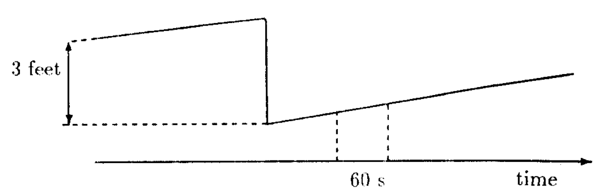

Figure 4. Pressure Sensor Drift

In summary, we now have two error-plagued values for the desired roll attitude indication, $z_1 \neq r$ from the observer and $r + d$ from the differential pressure sensors. In the first case, the value is inexact because of parametric uncertainties in the model, and in the latter case the value is corrupted by a significant drift. Clearly, it should be possible to extract a refined estimate of the roll attitude r by a judicious use of both of these measurements. To this end, we propose the following microprocessor-based filtering scheme.

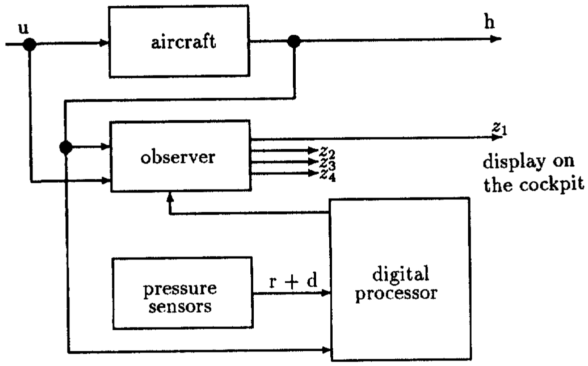

Figure 5: Proposed Filtering Scheme

5 Refined Roll Attitude: A Filtering Scheme

Consider the interconnection of the aircraft roll system, the state observer and the pressure sensors via the on-board digital processor shown in Figure 5.

5.1 Drift Update at $\dot{h} = 0$

The microprocessor system continuously scans the noiseless heading measurement, h, and determines the periods during which the heading is not changing, *i.e.* $\dot{h} = 0$. As indicated earlier, even the simplified linear model reproduces the well-known fact that the roll attitude r is zero whenever the aircraft heading remains steady. During periods of steady heading $\dot{h} = 0$, the output of the differential pressure system represents $(r + d)$ consists only of the drift voltage. The microprocessor continuously updates and stores the drift value during the $\dot{h} = 0$ periods and subtracts the updated drift value from the differential pressure indication of roll attitude.

5.2 Robust Asymptotic Observer Design

Using the nominal parameters given in Section 2 for the model of the lateral motion of a typical aircraft,

$$\begin{aligned} \alpha &= 3 \\ \beta &= 0.1 \\ \gamma &= 0.15 \\ \mu &= 120 \\ \nu &= 1 \end{aligned} \tag{7}$$

the observer equations become

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & K_1 & 0 \\ -\alpha\beta & -\alpha-\beta & K_2 & 0 \\ 0 & 0 & K_3 & 1 \\ \alpha & 0 & K_4 & -\gamma \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \mu \\ 0 \\ 0 \end{bmatrix} u - \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} h \tag{8}$$

For the choice

$$\begin{aligned} K_1 &= -6 \\ K_2 &= 1.6 \\ K_3 &= -5.4 \\ K_4 &= -9 \end{aligned} \tag{9}$$

the eigenvalues of the observer matrix are

$$\begin{aligned} \lambda_1 &= -1.75 + j0.5 \\ \lambda_/2 &= -1.75 - j0.5 \\ \lambda_3 &= -1.71 \\ \lambda_4 &= -3.41 \ . \end{aligned} \tag{10}$$

and are all still well to the left of the imaginary axis even for substantial variations about the nominal parameters. The observer states thus converge to values which differ from the state of the real system due to the aforementioned parametric uncertainties.

The digital processor now uses the drift-corrected pressure reading of the roll attitude to adjust the parameters of the observer as follows.

The steady state situation for the aircraft corresponding to the constant input u is

$$\begin{aligned} x_1 &= \text{constant} = \frac{\hat{\mu}}{\hat{\alpha}\hat{\beta}} u \\ x_2 &= 0 \\ x_3 &= \frac{\hat{\nu}}{\hat{\gamma}} x_1 t = h(t) \\ x_4 &= \frac{\hat{\nu}}{\hat{\gamma}} x_1 \end{aligned} \tag{11}$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\mu}, \hat{\nu}$ are the true unknown parameters. The above solution (11) corresponds to the airplane in a constant bank angle with a constant rate of change of the heading.

On the other hand, for constant u and linear $h(t)$, the observer converges to the steady state solution

$$\begin{aligned} z_1 &= \frac{\mu}{\alpha\beta} u \\ z_2 &= 0 \\ z_3 &= h(t) \\ z_4 &= \frac{\nu}{\gamma} z_1 \end{aligned} \tag{12}$$

The convergence time, according the observer eigenvalues, is about 2 seconds.

When the digital processor detects the situation of constant heading rate of change for over 2 seconds, it reads the drift-compenstated roll attitude r and the value of z_4 and adjusts the numerical value of α and γ in order to match left and right sides of the equalities

$$\left. \begin{aligned} \frac{\mu}{\alpha_1\beta} u = r \\ z_4 = \frac{\nu}{\gamma_1} r \end{aligned} \right\}$$

α_1 and γ_1 are the corrected values for α (previously =3) and γ (previously 0.15). The nominal values of μ, β, ν are preserved until another steady state situation with constant heading rate of change is detected, at which time parameters β and ν will be adjusted to the new values β_2 and ν_2 in order to match the equations. The process is repeated cyclically and the model parameters are all updated periodically yielding a refined observer estimate for the roll attitude angle. This roll angle is provided as a graphical artifical horizon presented in the cockpit on an appropriate CRT.

6 Conclusion

In this paper we have shown that system theoretic filtering of noisy roll attitude measurement can be used in conjunction with observer theory to produce reliable and improved estimates of the roll angle, even in the presence of model uncertainties. Further, work remains to be done in refining the parameter updating algorithm and in identifying the more sensitive parameters.

In the context of the overall study of the intergrated Navigational Instrument, current research is directed towards using differential pressure sensors located in the nose and the tail of the aircraft to measure pitch as well as the use of solid state sensors to detect heading. System theoretic refinement and cross-checking of these crude measurements presents an interesting challenge.

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